

WESTMINSTER SCHOOL
THE CHALLENGE 2021

MATHEMATICS II

Tuesday 27 April 2021

Time allowed: 1 hour 30 minutes

You will need a calculator for this paper.

All your working should be clearly shown.

You should attempt all the questions.

Please write in black or blue ink.

1 Joseph cycles 28 miles in 2 hours and 4 minutes. Arjun cycles 25 miles in 1 hour and 53 minutes. Which boy cycled faster on average? Show the working you do to decide.

2 Stephen is going to switch his home electricity company from Supplier A to Supplier B.

Supplier A charges 15.75 p per day, plus 13.44 p per kilowatt-hour of electricity.

Supplier B charges 11.32 p per day, plus a certain rate per kilowatt-hour of electricity.

Stephen knows that his annual electricity usage (in a year of 365 days) is 2270 kilowatt-hours. He calculates that he will save £65.20 per year by switching supplier. Find the rate that Supplier B charges per kilowatt-hour of electricity.

3 The bar is a unit of pressure equal to 100 000 pascals.
1 pascal is a pressure of 1 newton per square metre.

An inscription on the side of my car tyre reads “recommended pressure 2.3 bar”. My pump has a gauge that shows pressure in pound-force per square inch (psi). What should the gauge on my pump read when the tyre is correctly inflated?

[Conversions: 1 pound-force \approx 4.448 newtons ; 1 inch = 2.54 cm.]

4 a i What is the result of adding $\frac{x+1}{2}$ and $\frac{1}{4}(x-3)$?

ii By what would you need to multiply $\frac{2x}{y}$ to make $\frac{y}{2x}$?

b Make T the subject of

$$\frac{3}{1-T^3} = P.$$

c Solve the equation

$$\frac{2}{3}x = \frac{1}{4} - \frac{5}{6}(7-x).$$

5 A gentleman bought a horse and carriage for £750. He later sold the horse at a profit of 5% and the carriage at a profit of 20%. Overall, his profit was 16%. Find the price the gentleman originally paid for the carriage.

6 The combined ages of a mother and father are six times the combined ages of their children. Two years ago, their combined ages were ten times the combined ages of their children. In six years' time, their combined ages will be three times the combined ages of their children. How many children do they have?

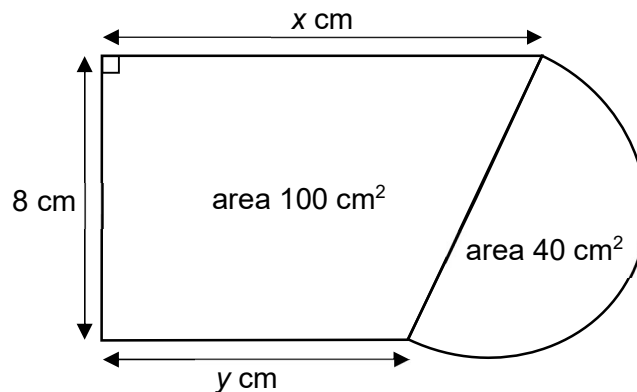
- 7 a The number of cases of an infectious disease grows by 6% each day. If there are 33 502 cases on March 29th, how many cases were there on March 1st?
- b The number of hairs on an aging gentleman's head decreases by a fixed percentage each year. On January 1st 2016, the gentleman had 85 000 hairs on his head. On January 1st 2021, he was left with just 24 000. How many did he have on January 1st 2019?

- 8 The express train leaves London at 1 pm and arrives in Manchester at 3:30 pm. The slow train leaves Manchester at 11 am and arrives in London at 3 pm. If each train travels at a constant speed, at what time do they pass each other? Give your answer to the nearest minute.

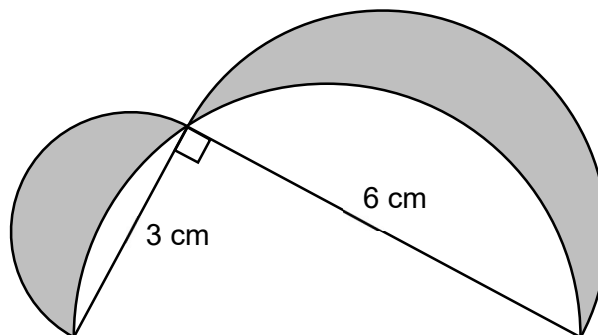
- 9 Two large vessels, P and Q, contain mixtures of water and wine. In vessel P there is three times as much wine as water. In vessel Q there is five times as much water as wine.

Find how much liquid should be taken from each vessel in order to fill a third vessel of capacity 19.25 litres with a 50:50 mixture of wine and water.

- 10 The diagram shows a trapezium and a semicircle. The area of the trapezium is 100 cm^2 and the area of the semicircle is 40 cm^2 . Find x and y .



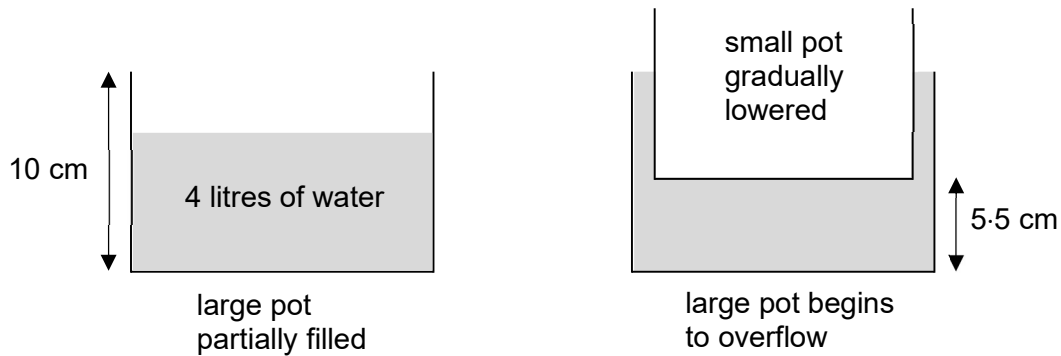
- 11 The diagram below shows three semicircles and a right-angled triangle. Find the total shaded area.



- 12** I have a set of two cylindrical but differently sized cooking pots. The large pot is 10 cm in height. The radius of the small pot is nine-tenths that of the large pot.

The large pot is partially filled with 4 litres of water, and I gradually lower the empty small pot into the water. The large pot begins to overflow when the base of the small pot is 5.5 cm above the base of the large pot.

Find the radius of the large pot correct to two decimal places.



- 13** This question concerns lists of the digits 1 to 9, not necessarily in numerical order, that use all nine digits exactly once each.

Such a list is said to be an “ n -left-sum-list” if the sum of the leftmost X digits is n , where X is the leftmost digit in the list.

For instance,

5 4 1 3 8 2 6 7 9

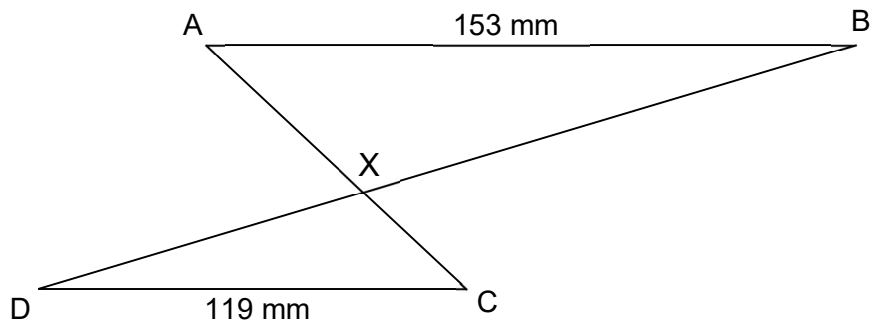
is a 21-left-sum-list, because the leftmost digit in the list is 5, and the sum of the leftmost 5 digits is $5 + 4 + 1 + 3 + 8$, which equals 21.

- Find a 16-left-sum-list.
- Explain why the leftmost digit of a 30-left-sum-list cannot be 4 or 8.
- What are the possibilities for the leftmost digit of a 17-left-sum-list? Justify your answer.

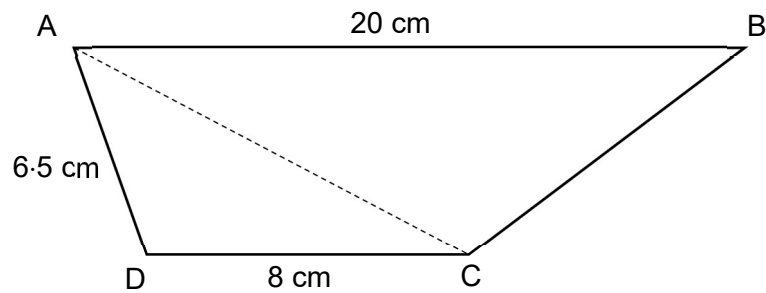
A list is said to be an “ m -right-sum-list” if the sum of the rightmost Y digits is m , where Y is the rightmost digit in the list.

- A certain list is both a 17-left-sum-list and a 17-right-sum list. Explain why the list must start with 3 and end with 4, or vice-versa.

- 14 a** In the diagram, AB is parallel to CD, and AXC and BXD are straight lines. Length BD is 240 mm and length CX is 56 mm.



- i** Explain why triangle ABX is similar to triangle CDX.
 - ii** Find length BX.
 - iii** Show that angle AXB is a right angle.
- b** In the diagram, ABCD is a trapezium and angle ADC is equal to angle ACB. Find the perimeter of the trapezium correct to two decimal places.



END OF QUESTIONS