WESTMINSTER SCHOOL
THE CHALLENGE 2021

MATHEMATICS III

Wednesday 28 April 2021

Time allowed: 1 hour 30 minutes

You may not use a calculator for this paper.

All your working should be clearly shown.

You should attempt all the questions.

Please write in black or blue ink.

1   It takes 54 minutes for 242 cats to drink 396 pints of milk.
How long does it take 195 cats to drink 325 pints of milk?

2   Solve the following simultaneous equations.

$$x = 3(4y - x + 2)$$
$$y = 5(x - y + 3)$$

3   Four hammers, four spanners and four screwdrivers together weigh 3·68 kg.
Seven hammers, five spanners and six screwdrivers together weigh 5·81 kg.
How much heavier than a spanner is a hammer?

4   In the diagram below, the lines FA and DCB are parallel. Several of the angles are
shown.
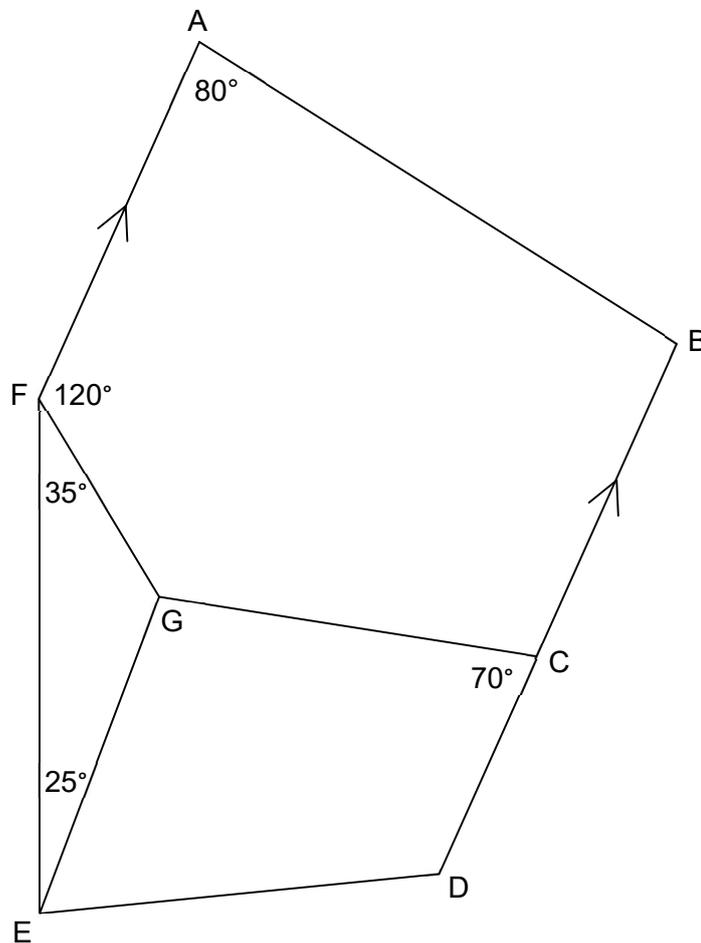
a   Determine the size of angle FGC, justifying each step of your argument clearly.

b   Prove that lines EG and DC are parallel.

5 In my cellar, I have a wine collection containing red and white wine only. Sadly, some of the wine is corked and therefore undrinkable; the rest is drinkable.

There is five times as much drinkable white wine as corked red wine.
There is eight times as much drinkable red wine as corked white wine.
There is six times as much drinkable wine as corked wine.

In what ratio does my cellar contain red and white wine?

6 Sorcery cards are sold in packs of 50–100 cards. In particular, regular packs all contain M cards, and bumper packs all contain N cards. It is possible to buy 882 cards either with regular packs entirely or with bumper packs entirely, and there is no smaller number of cards for which this is possible. Find M and N.

7 I have a vase and a metal ornament, both in the shape of a cylinder. The base radius of the ornament is smaller than the base radius of the vase. The vase contains water with a depth of 6 cm.

When I place the ornament in the vase, as shown, so that the base of the ornament rests flat against the base of the vase, the water level rises to a depth of 8·8 cm, without overflowing. The ornament is not submerged.

6 cm

8·8 cm
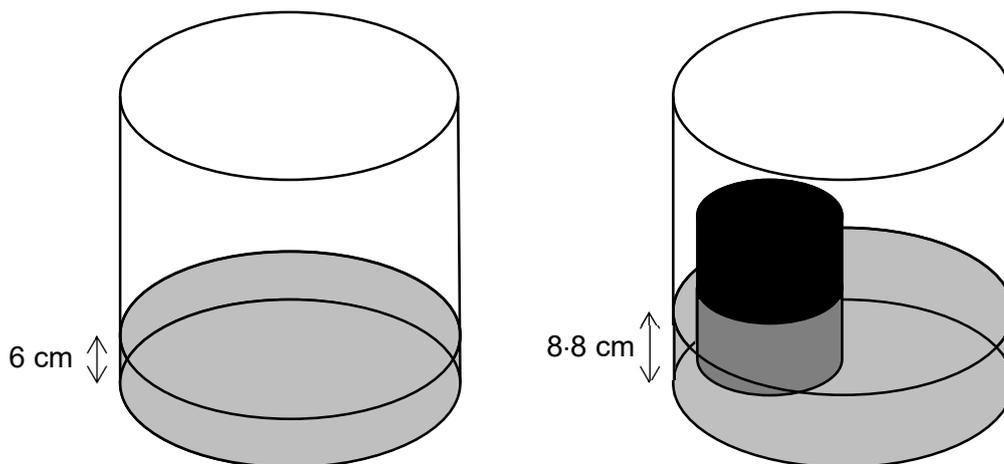
I have a second ornament, identical to the first. I place both into the water, with their bases resting flat against the base of the vase. The ornaments are not submerged, and the water does not overflow. To what depth will the water rise?

8 At the ball bearing factory, some ball bearings are defective because they are hollow. Some ball bearings are defective because they are misshapen. Some are defective for both reasons, but there are no other reasons for a ball bearing to be defective.

22% of ball bearings are defective.
25% of hollow ball bearings are also misshapen.
40% of misshapen ball bearings are also hollow.

What percentage of ball bearings are hollow?

9 The partial diagram below shows a regular hexagon, a square, and a third regular shape. How many sides does the third shape have?
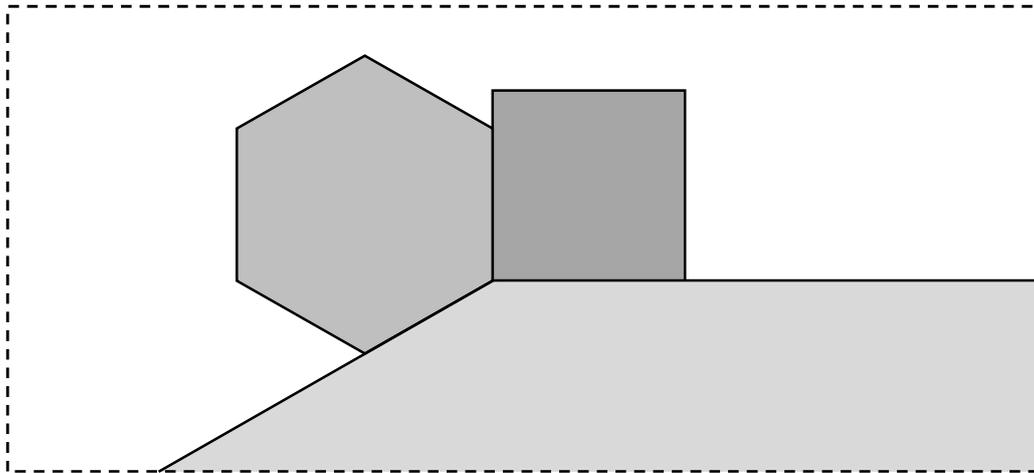
10 A novice lawn-shearer, a journeyman lawn-shearer and a master lawn-shearer are preparing to trim a lawn with scissors.

If the novice and the journeyman work together, they will take 2 hours 48 minutes.
If the novice and the master work together, they will take exactly 2 hours.
If the journeyman and the master work together, they will take 1 hour 45 minutes.

How long will it take if all three of them work together?

11 For standard letters, first-class stamps cost 85p and second-class stamps cost 66p.

a If I spend £36·23 to post fifty standard letters, how many are sent first-class?

b There is only one way to spend £17·09 to post standard letters. What is it?

12 Four runners, in two pairs, stand together on a standard 400m athletic track. They are about to run a 2×400m relay race, in which the first runner from each pair runs a full lap before reaching his partner, who then starts his own lap. One pair is Tweedledum and Tweedledee, in that order. The other pair is Hesperus and Phosphorus, in that order.

Each runner has a constant speed while running:
- Tweedledum runs 20% slower than Hesperus.
- Tweedledee runs 20% faster than Phosphorus.
- Hesperus and Phosphorus run at the same speed as each other.

Which team will win the race, and by what distance?

13 **a** Multiply out and simplify the expression $(x+y)(x-y)$.

In the diagram below, the shaded region forms an *annulus*, created by cutting a circular disc out of a larger one.
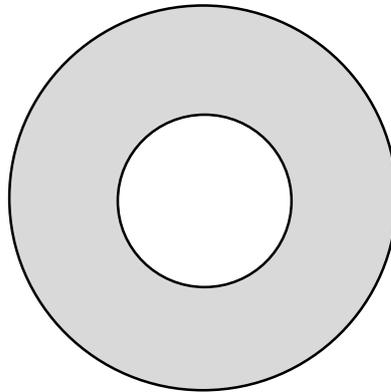
b Suppose that the outer radius is 6·7 cm and the inner radius is 3·3 cm.
Use your answer to part **a** to show that the area of the annulus is 34π cm^2.

c Find another pair of radii for which the area of the annulus is 34π cm^2.

14 Fergal and Joaquin have two bags, A and B, each containing some wooden cubes of identical size.

a Fergal tosses some cubes from bag A onto a table, leaving some in the bag. He notices that there is a square number of cubes on the table, and he arranges them into a square shape.

Fergal says: "If we'd used *all* the cubes instead, we could have made a rectangle which is 12 cubes longer and 5 cubes wider than the square."

Joaquin says: "True, but we could also have made a rectangle which is 15 cubes longer and 3 cubes wider than the square."

Determine the number of cubes originally in bag A.

b Joaquin tosses some cubes from bag B onto a different table, leaving some in the bag. He arranges these cubes into a rectangular shape.

Fergal says: "If we'd used *all* the cubes instead, we could have made a rectangle which is 3 cubes longer and 3 cubes wider than the original rectangle."

Joaquin says: "True, but we could also have made a rectangle which is 2 cubes *shorter* and 7 cubes wider than the original rectangle."

Given that bag B originally contains more cubes than bag A, what is the smallest possible number of cubes originally in bag B?

END OF QUESTIONS