WESTMINSTER SCHOOL
THE CHALLENGE 2024

# MATHEMATICS II

Tuesday 30 April 2024

Time allowed: 1 hour 30 minutes

You will need a calculator for this paper.

All your working should be clearly shown.

You should attempt all the questions.

Please write in black or blue ink.

Write your answers in the answer booklet provided.

**1** **a** **i** Subtract $\frac{1}{2}x - 1$ from $\frac{1}{2}(x-1)$ and simplify your answer.

**ii** By what must you divide $28a$ to get $7a$ ?

**b** Tom correctly solved a pair of simultaneous equations in his maths homework, but parts of the question and his answer have since been obscured by ink blots.

*question:* Solve the simultaneous equations

$$\frac{5}{2}x - \frac{3}{4}y = \frac{8}{3}$$

$$\blacksquare x + \frac{1}{2}y = \frac{1}{12}$$

*Tom's correct answer:* $x = \frac{11}{12}$ and $y = \blacksquare$

Find the numbers hiding under the ink blots.

**2** The density of steel is 7.85 grams per cubic centimetre. Four thousand identical spherical steel ball bearings together weigh 1.4 kilograms. Find the radius of each ball bearing in millimetres.

[A sphere of radius $r$ centimetres has volume $\frac{4}{3}\pi r^3$ cubic centimetres.]

**3** In "old money", each pound was divided into 20 shillings and each shilling was divided into 12 pence.

In February 1970, a jeweller sold one hundred gold bracelets. The first sixty-eight bracelets were sold at 1 pound, 7 shillings and 6 pence each, but the remainder were sold at a discounted price. His total takings for gold bracelets in February 1970 were 124 pounds and 6 shillings.

What percentage discount did the jeweller offer, and what was the discounted selling price (in pounds, shillings and pence)?

**4** **a** Arif is 14 years old. His pocket money now is 48% greater than it was when he was 12. When he was 13, his pocket money was £19.20 per week, which was 28% greater than it was when he was 12. How much is it now?

**b** Billy has a barrel full of chocolate drops. Each week, he eats $n$% of the chocolate drops remaining in the barrel at the start of that week. At the end of the seventh week, he finds that only 10% of the original quantity of chocolate drops is left. What is the value of $n$?

**5** Sequences of numbers are produced using the rule "*multiply by p, then subtract q*". For example, if $p = 2$, $q = 3$ and the first term is 6, then the sequence would begin

6 9 15 27 51 99 etc.

**a** If $p = 1.5$, $q = 0.5$ and the first term is 17, write out the first five terms.

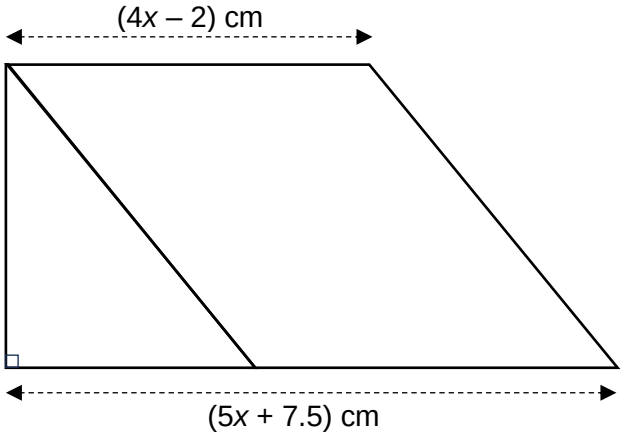
**b** If, instead, the sequence begins 20, 45, 170,… , what are the values of $p$ and $q$?

**c** If, instead, the fourth, fifth and sixth terms are 48, 142.5 and 426 respectively:

**i** What is the first term?

**ii** Explain why the $2024^{th}$ term will be a whole number and find its last digit. Justify your answer briefly.

**6** In the diagram, the large trapezium is made up of a right-angled triangle and a parallelogram. The area of the large trapezium is 338.1 $cm^2$; this area is divided between the right-angled triangle and the parallelogram in the ratio 7 : 16. Find the perimeter of the large trapezium.



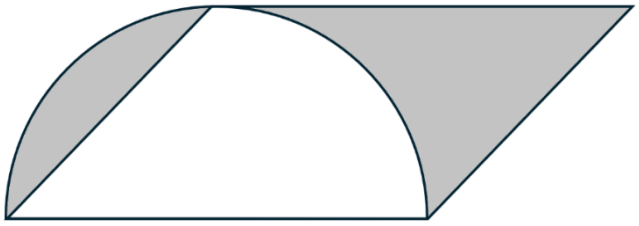

**7** After a Scout Jamboree, 1003 Scouts travelled home by coach or minibus.
Each coach carried 53 Scouts and each minibus carried 13 Scouts.
There were $x$ coaches and $y$ minibuses. Find the possible pairs of values of $x$ and $y$.

**8** Mr Tiring walks to and from work on Mondays, Tuesdays and Wednesdays. On Tuesdays, his speed is two-thirds of his speed on Mondays. On Wednesdays, his speed is three-quarters of his speed on Tuesdays. On Thursdays and Fridays, feeling exhausted, he takes the bus along the same route that he walks. The bus travels at a speed of 7 miles per hour.

One quarter of Mr Tiring's weekly commuting time is spent on the bus. How fast does he walk on Mondays?

**TURN OVER**

**9** The logo in the diagram consists of an overlapping semicircle and parallelogram. To the nearest whole number, what percentage of the logo's area is shaded?



**10** Andrew is solving a puzzle in a mathematical magazine. To complete the puzzle, he must fill in every cell of the grid below with one of the digits 1, 2, 3, 4 and 5, in such a way that the *Puzzle Property* is always true:

| | | | | |
|---|---|---|---|---|
| *Puzzle Property:* | whenever | digit *d* | appears in | column *x*, row *y*, |
| | then | digit *x* | appears in | column *d*, row *y*, |
| | and also | digit *y* | appears in | column *x*, row *d*. |

The **bold** digits were given as part of the puzzle.

| | col 1 | col 2 | col 3 | col 4 | col 5 |
|---|---|---|---|---|---|
| row 1 | **2** | | | | |
| row 2 | | | | | **2** |
| row 3 | | | **1** | | |
| row 4 | **5** | | | | ***1*** |
| row 5 | | | **5** | | |

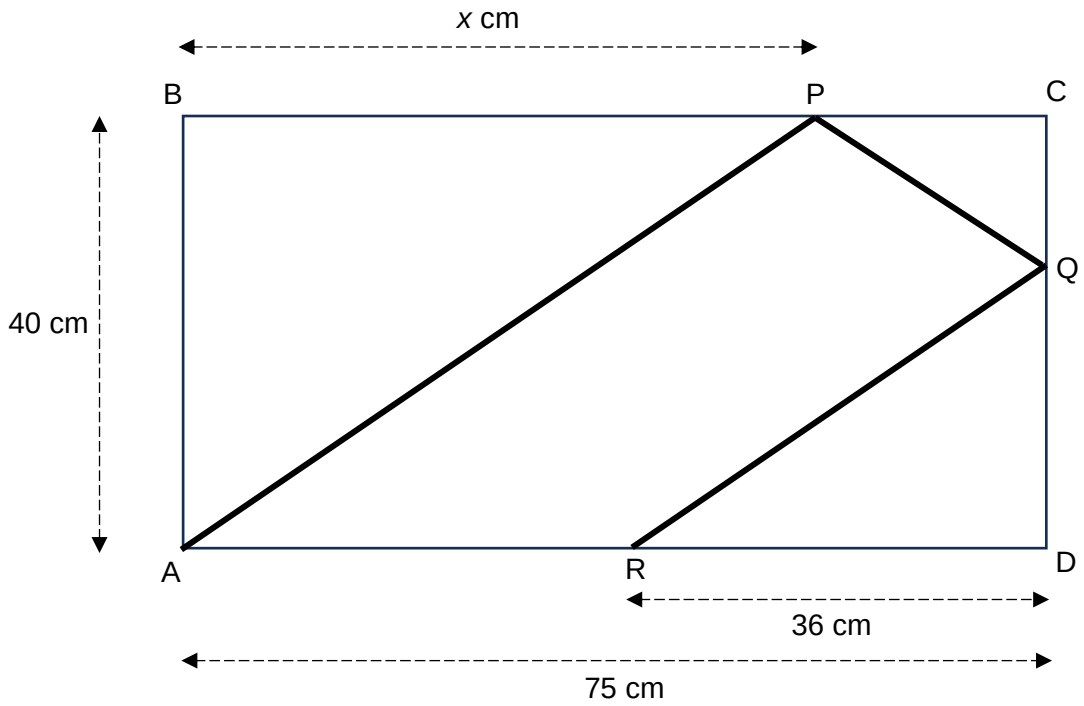
**a** Andrew has used the *Puzzle Property* to write digit 1 in column 5, row 4.
What digit can he write in column 1, row 5?
What digits can he subsequently write in

- column 5, row 1
- column 4, row 5
- column 4, row 1 ?

**b** Prove that the *Puzzle Property* forbids the same digit from appearing twice in any row or column.

**c** Make a copy of the grid in your answer booklet and complete the puzzle.

**11** A laser beam is fired from point A, which is at one corner of a rectangular mirror box ABCD. It reflects off the mirror walls at points P and Q, as shown by the heavy line in the diagram, before striking a receptor at R. Some lengths are shown on the diagram.




**i** Explain why triangles ABP, QCP and QDR are all mathematically similar.
**ii** Hence find *x*.

**12** **a** The angles in an isosceles triangle are 45°, 45° and 90°, and the length of the equal sides is 1. What is the length of the remaining side?

**b** Grace has two identical cardboard rectangles, which she overlaps to make the polygon shown in the left-hand diagram. She finds the area and perimeter of the polygon.

Later, Grace moves one of the rectangles so that it makes a 45° angle with the other, as shown in the right-hand diagram. Doing this reduces the perimeter of the polygon by 5 cm. By how much does it reduce the area?

**END OF QUESTIONS**

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