WESTMINSTER SCHOOL

The Challenge (Specialism)

Specimen Paper First Examination 2027

MATHEMATICS

Time: 60 minutes

Instructions to candidates:

- You may not use a calculator for this paper.
- All your working should be clearly shown.
- You should attempt all the questions.
- Please write in black or blue ink.
- Write your answers in the answer booklet provided.

1 Silk ribbon costs £1.05 per metre, and satin ribbon costs £0.85 per metre. I buy 50 m of ribbon, which costs £48.74 in total. How much silk ribbon did I buy?

2 What day of the week will it be, exactly 10 000 hours after midday today?

[N.B. This question originally appeared in a paper taken on a Wednesday.]

3 If a set of twelve values has a median of 41.7 and a range of 8.4, what is the greatest possible value of the mean? Give your answer as a decimal.

4 Frog and Toad are riding experimental vehicles powered by canisters of fuel. Frog's vehicle can travel 13 miles per canister, and Toad's vehicle can travel 20 miles per canister. All canisters are identical and can be inserted into either vehicle, but each canister can only be used on one vehicle once inserted, and the fuel content of a canister cannot otherwise be extracted or shared between vehicles.

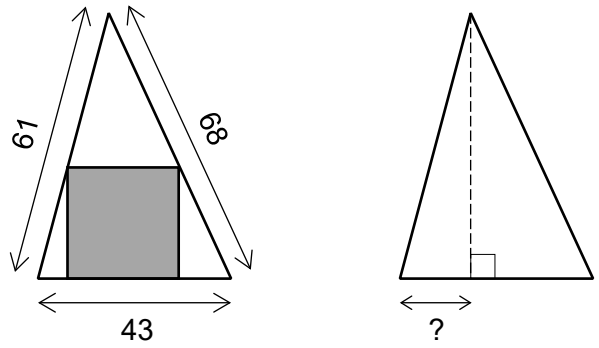
Frog and Toad have a total of 24 canisters between them, and they want to travel together as far as they can, starting from the same point. How far can they go?

5 **a** Expand and simplify the expression $(b + 11)(b - 11)$.

b Use $b = 989.001$ to find 989.001^2 as an exact decimal.

6 A standard indoor athletics track is a loop of length 200 m. Achilles and Brian are racing a distance of 3 000 m on such a track. They start at the same place at the same time, run anticlockwise around the track at their own constant speed, and each runner stops when he has run 3 000 m. Achilles runs at 6.2 m/s and Brian runs at 4.9 m/s. How many times does Achilles overtake Brian on the track before winning the race?

7 Shown below is a shaded square inside a triangle, and an exact copy of the triangle.



a Find the length marked in the second diagram.

b Find the side length of the shaded square. Give your answer as a mixed fraction.

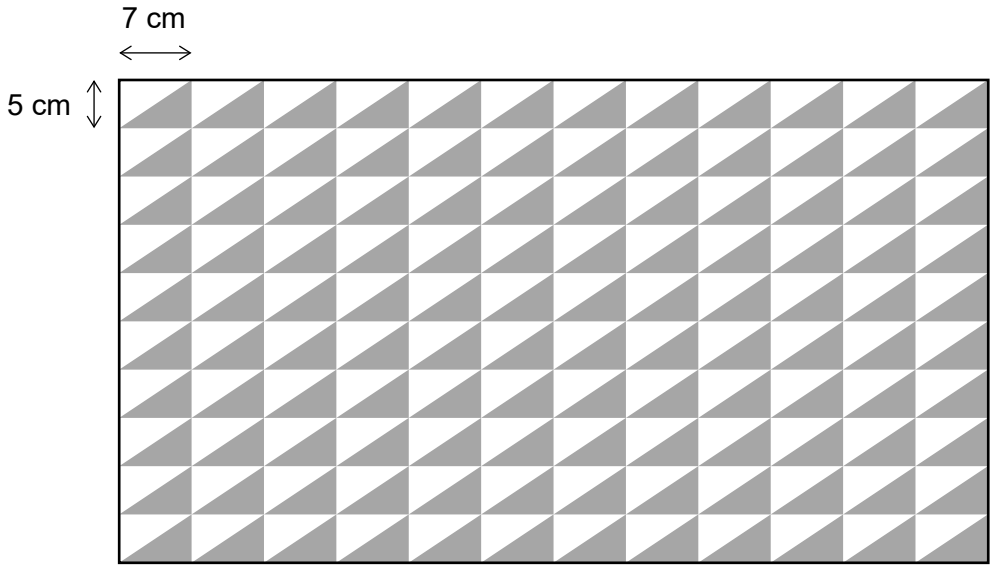
8 Some of my pet stingrays have spots on their backs, some have spots on their tummies, and some have both. Half of my stingrays have no spots at all. Among the stingrays with spots on their backs, one-seventh also have spots on their tummies. Among the stingrays with spots on one side only, three-fifths have spots on their backs. What is the smallest number of pet stingrays that I could possibly have?

9 The sequence below begins with 333 333, in which all digits are the same. Each subsequent term is 17 094 greater than the previous one.

333 333, 350 427, 367 521, 384 615, 401 709, …

Where in the sequence is the next term in which all digits are the same?

10 A sheet of wrapping paper is decorated with an alternating pattern of white and silver triangles, and is illustrated below.



A square of side length L cm is cut from the top-left corner of the sheet. The total silver area inside the square is A cm².

a If $L = 28$, find A as a decimal.

b If, instead, $A = 626$, find L as a decimal.

END OF QUESTIONS

ANSWERS

1 Silk ribbon costs £1.05 per metre, and satin ribbon costs £0.85 per metre. I buy 50 m of ribbon, which costs £48.74 in total. How much silk ribbon did I buy?

$1.05x + 0.85(50 - x) = 48.74$

31.2 m

2 What day of the week will it be, exactly 10 000 hours after midday today?

[N.B. This question originally appeared in a paper taken on a Wednesday.]

10000 ÷ 24 = 416 remainder 16 hours. 416 ÷ 7 has remainder 3 days. But 16 hours after midday takes us into the next day, so it's +4 days within the week.

Sunday

3 If a set of twelve values has a median of 41.7 and a range of 8.4, what is the greatest possible value of the mean? Give your answer as a decimal.

With a bit of thought to confirm that it's better to have 41.7 as the sixth and seventh values rather than the midpoint of two distinct numbers, the answer is $\frac{7\times41.7+5\times(41.7+8.4)}{12}$

45.2

4 Frog and Toad are riding experimental vehicles powered by canisters of fuel. Frog's vehicle can travel 13 miles per canister, and Toad's vehicle can travel 20 miles per canister. All canisters are identical and can be inserted into either vehicle, but each canister can only be used on one vehicle once inserted, and the fuel content of a canister cannot otherwise be extracted or shared between vehicles.

Frog and Toad have a total of 24 canisters between them, and they want to travel together as far as they can, starting from the same point. How far can they go?

Solving $13x = 20(24 - x)$ gives $x = 14\frac{6}{11}$ which is not an integer. Rounding to the nearest integer [$x = 15$] suggests an answer of 180 miles, but the optimum solution is in fact the other nearby integer [$x = 14$] giving an answer of 182 miles.

5 **a** Expand and simplify the expression (*b* + 11)(*b* − 11).

b² − 121

b Use *b* = 989.001 to find 989.001² as an exact decimal.

$989.001^2 = 1000.001 \times 978.001 + 121 = 978001.978001 + 121$

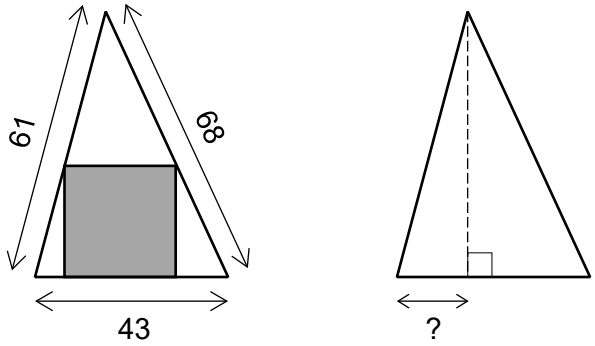
978122.978001

6 A standard indoor athletics track is a loop of length 200 m. Achilles and Brian are racing a distance of 3 000 m on such a track. They start at the same place at the same time, run anticlockwise around the track at their own constant speed, and each runner stops when he has run 3 000 m. Achilles runs at 6.2 m/s and Brian runs at 4.9 m/s. How many times does Achilles overtake Brian on the track before winning the race?

Achilles overtakes Brian every time his distance travelled exceeds Brian's by a multiple of 200 m. When Achilles finishes his race and stops running, Brian will have run $3000 \times \frac{4.9}{6.2} \approx 2371$ m. In a non-calculator context, only the first two digits are needed, because this establishes that the distance gap at that moment is strictly between 3×200 m and 4×200 m, so Achilles will have overtaken Brian 3 times but not 4.

Three times.

7 Shown below is a shaded square inside a triangle, and an exact copy of the triangle.



a Find the length marked in the second diagram.

$61^2 - x^2 = 68^2 - (43 - x)^2$

$[x =]11$

b Find the side length of the shaded square. Give your answer as a mixed fraction.

The height of the triangle is 60. Use $\frac{\text{height}}{\text{base}}$ with similar triangles to get $\frac{60}{43} = \frac{60-y}{y}$

$[y =]25\frac{5}{103}$

8 Some of my pet stingrays have spots on their backs, some have spots on their tummies, and some have both. Half of my stingrays have no spots at all. Among the stingrays with spots on their backs, one-seventh also have spots on their tummies. Among the stingrays with spots on one side only, three-fifths have spots on their backs. What is the smallest number of pet stingrays that I could possibly have?

(Ratio:)	Tummy	Not tummy
Back	1	6
Not back	4	11

22, since all categories must contain a whole number of stingrays.

9 The sequence below begins with 333 333, in which all digits are the same. Each subsequent term is 17 094 greater than the previous one.

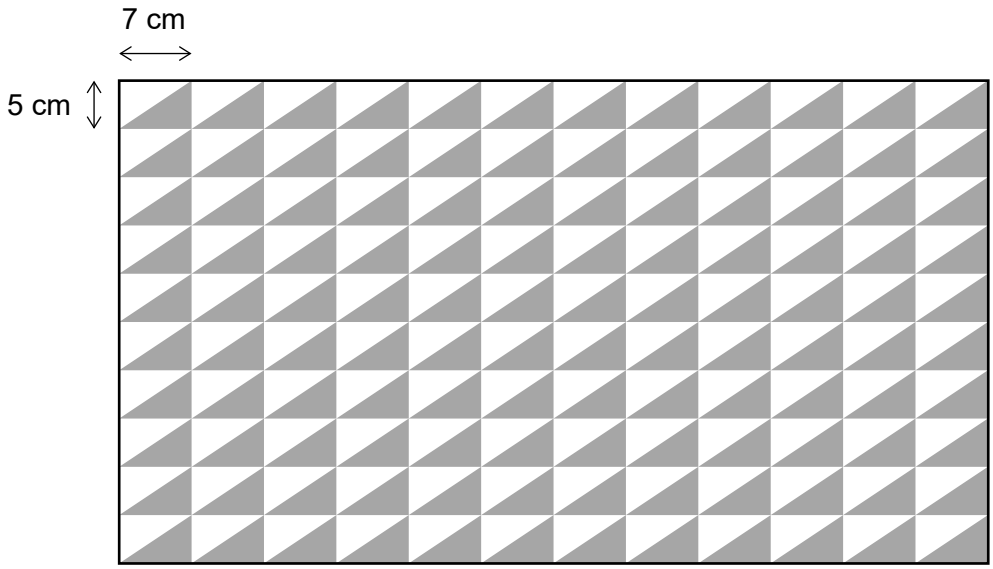
333 333, 350 427, 367 521, 384 615, 401 709, …

Where in the sequence is the next term in which all digits are the same?

It's not at all obvious that the next term with all digits the same will have precisely six digits, but it's worth exploring. You can find HCF(111111, 17094) by working from both ends: 17094 is even, so consider 17094 ÷ 2 = 8547, while 111111 is divisible by 3, so consider 111111 ÷ 3 = 37037. Then consider 8547 ÷ 3 = 2849, which is clearly divisible by seven. Meanwhile 37037 is clearly divisible by 37, etc.

The 14th term (which is 555 555).

10 A sheet of wrapping paper is decorated with an alternating pattern of white and silver triangles, and is illustrated below.



A square of side length L cm is cut from the top-left corner of the sheet. The total silver area inside the square is A cm².

a If $L = 28$, find A as a decimal.

28 is a multiple of 7 but not 5, so the square in question captures a whole number of columns but not a whole number of rows of triangles. There is some difficulty in finding the area of the (four) small triangles captured in the bottom partial row, but it helps to spot that they are similar to the whole triangles.

$A = 375.2$

b If, instead, $A = 626$, find L as a decimal.

626 cm² is equal to the total area of 35 triangles plus a remainder, so our square will be slightly bigger than a square that is exactly five columns wide and seven rows deep. The remainder of 13.5 cm² needs to be equal to the total area of the five small triangles along the bottom and seven small(er) triangles along the side, etc.

$L = 36.5$

[N.B. When this question was asked in a recent Challenge paper, none of the candidates got the final answer right. If you were able to get it right in a reasonable time and without a calculator, then you are doing very well and you should certainly consider taking the Maths Specialism paper.]

END OF QUESTIONS